

Coherent states of a particle in a magnetic field and the Stieltjes moment problem

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Abstract

A solution to a version of the Stieltjes moment problem is presented. Using this solution, we construct a family of coherent states of a charged particle in a uniform magnetic field. We prove that these states form an overcomplete set that is normalized and resolves the unity. By the help of these coherent states we construct the Fock-Bergmann representation related to the particle quantization. This quantization procedure takes into account a circle topology of the classical motion.

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1 Introduction

Constructing semiclassical or “classical-like” states in quantum mechanics is in general an open problem. For systems with quadratic Hamiltonians there exists a well-known procedure to construct the so-called coherent states (CS, or Glauber-Klauder-Sudarshan, or standard CS), which usually are accepted as quantum states that behave like their classical counterpart, see e.g. [1, 2, 3, 4]. The CS are widely and fruitfully being utilized in different areas of theoretical physics. The standard CS turned out to be orbits of the Heisenberg-Weyl group. This observation allowed one to formulate by analogy some general definition of CS for any Lie group [5, 6, 7, 8] as orbits of the group factorized with respect to a stationary subgroup. There exists a connection between the CS and the quantization of classical systems, in particular, systems with a curved phase space, see e.g. [9, 10].

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In [11] a modified approach to constructing semiclassical or coherent states (we call them also CS) was proposed. A technical realization of the approach recipes depends on each concrete case, in particular, a principal one is the problem of proving the resolution of the unity by the constructed CS. In the present article, we construct coherent states for a charged particle in a constant and uniform magnetic field closely by following the approach of [11], more exactly a squeezed version of these states. By solving a specific version of the Stieltjes moment problem we make explicit the resolution of the unity of the constructed CS and so become able to perform the Berezin-Klauder-Toeplitz or, more simply CS, quantization of the complex plane. A generalization of the obtained results to a model on a non-commutative plane will be the topic of a further work.

As far as physical applications are concerned, the resolution of the unity by CS is fundamental for the analysis, or decomposition, of states in the Hilbert space of the problem, or of operators acting on this space. In particular, it allows for a “classical” reading of quantum dynamical systems, in Schrödinger representation, through the time behavior of mean values of quantum observables in coherent states. Nice illustrations of this approach are provided by Perelomov in [5]. It was precisely this symbolic formulation that enabled Glauber and others to treat a quantized boson or fermion field like a classical field, particularly for computing correlation functions or other quantities of statistical physics, such as partition functions and derived quantities.

2 Coherent states of a particle in magnetic field

Consider a charged particle with charge e and mass μ placed in a uniform and constant magnetic field of magnitude B in the z -direction. The motion of the particle in a plane perpendicular to the magnetic field can be described by the quantum Hamiltonian ($c = \hbar = 1$)

$$H = \frac{1}{2\mu} (P_1^2 + P_2^2) , \quad P_i = p_i - eA_i , \quad A_i = -\frac{B}{2}\varepsilon_{ij}x^j , \quad i, j = 1, 2, \quad (1)$$

where x^i and p_i are canonical operators of coordinates and momenta of the particle and ε_{ij} ($\varepsilon_{12} = 1$) is the Levi-Civita tensor. It is useful to introduce operators x_0^i , which are integrals of motion and correspond to the orbit center coordinates,

$$x_0^i = x^i + \frac{1}{\mu\omega}\varepsilon^{ij}P_j , \quad \omega = \frac{eB}{\mu} ,$$

and also the angular momentum operator of the relative motion J , which in the present case is just proportional to the Hamiltonian,

$$J = -\frac{1}{\omega}H = \frac{1}{2} (r^1P_2 - r^2P_1) , \quad r^i = x^i - x_0^i .$$

Two independent Weyl-Heisenberg algebras underlie the symmetries and the integrability of the model. The first one concerns the operators $r_{0\pm} = x_0^1 \pm ix_0^2$

that obey $[r_{0-}, r_{0+}] = 2/\mu\omega$. The second one concerns the relative motion operators r^i , $r_{\pm} = r^1 \pm ir^2 = -[P_2 \mp iP_1]/\mu\omega$ with $[r_+, r_-] = 2/\mu\omega$. They allow one to construct a Fock space with orthonormal basis $\{|m, n\rangle\}$ obtained by repeated actions of the normalized raising operators:

$$\sqrt{\mu\omega/2}r_{0+}|m, n\rangle = \sqrt{m+1}|m+1, n\rangle, \quad \sqrt{\mu\omega/2}r_{0-}|m, n\rangle = \sqrt{n+1}|m, n+1\rangle.$$

Like in [11], the CS $|z_0, \zeta\rangle$ are introduced as solutions of the eigenvalue problems

$$r_{0-}|z_0, \zeta\rangle = z_0|z_0, \zeta\rangle, \quad Z|z_0, \zeta\rangle = \zeta|z_0, \zeta\rangle, \quad z_0, \zeta \in \mathbb{C}, \quad (2)$$

with $Z = e^{-J+\frac{1}{2}}r_+$. The commutation relations $[J, r_{\pm}] = \pm r_{\pm}$ reproduce the appropriate algebra to study the circular motion, see [12]. These normalized CS are tensor product of the state $|\zeta\rangle$, that is an eigenvector of Z with the standard CS $|z_0\rangle$. They read in terms of the Fock basis,

$$|z_0, \zeta\rangle = \frac{1}{\sqrt{\mathcal{N}(z_0, \zeta)}} \sum_{m,n=0}^{\infty} \left(\frac{\mu\omega}{2}\right)^{\frac{m+n}{2}} \frac{z_0^m}{\sqrt{m!}} \frac{\zeta^n e^{-\frac{1}{2}(n+\frac{1}{2})^2}}{\sqrt{n!}} |m, n\rangle, \quad (3)$$

where \mathcal{N} stands for a normalization factor.

One can easily see that the time evolution of the CS states is only reduced to the classical time evolution of the parameter ζ . Indeed, using the relations $[r_{0\pm}, H] = 0$ and $e^{itH}Ze^{-itH} = e^{-i\omega t}Z$, and applying the evolution operator $U(t) = \exp(-iHt)$ to the state $|z_0, \zeta\rangle$ we obtain

$$U(t)|z_0, \zeta\rangle = |z_0, \zeta(t)\rangle, \quad \zeta(t) = \zeta \exp(-i\omega t).$$

In order to play their role of “classical-between-quantal” bridge, the states $|z_0, \zeta\rangle$ have to resolve the identity in the above Fock-Hilbert space. This specific (over-)completeness can be proved by resolving the identity in the corresponding Fock-Bergmann representation. As was stated in [11], the task is equivalent to solving the following moment problem

$$\int_0^{\infty} t^n \varpi_q(t) dt = n!q \frac{n(n+1)}{2}, \quad q = e^2, \quad (4)$$

for an unknown weight function $\varpi(t)$. Let us generalize the above problem and, consequently, obtain a squeezed version of the CS (2), by introducing the following displacement operator

$$Z_{\lambda} = \exp\left[\frac{\lambda}{2}\left(\frac{1}{2} - J\right)\right] r_+. \quad (5)$$

This operator coincides with Z from (2) for $\lambda = 2$, and with just r_+ for $\lambda = 0$ (or $q = 1$), i.e., the case where we have the tensor product of standard coherent states, called in this context the Malkin-Man’ko CS [13]. For an arbitrary λ the operator Z_{λ} controls the dispersion relations of the angular moment and of the

position operators. In this case, the construction of the resolution of identity from the eigenstates of the above operator, and consequently the proof that they form an (over-)complete set, is equivalent to solving the moment problem of the form

$$\int_0^\infty t^n \varpi_q(t) dt = n! q^{\frac{n(n+1)}{2}}, \quad q \equiv e^\lambda, \quad \lambda \geq 0,$$

for some unknown weight function $\varpi_q(t)$. Below, we find ϖ_q for an arbitrary $q \in [1, \infty)$ and deal with the corresponding CS as the eigenstates of the operator Z_λ .

3 Solving Stieltjes moment problem

Let us consider the classical phase space $\mathbb{C}^2 = \{\mathbf{x} = (z, \zeta), z \in \mathbb{C}, \zeta \in \mathbb{C}\}$ provided with the measure:

$$\mu(d\mathbf{x}) = e^{-|z|^2} \frac{d^2 z}{\pi} \varpi_q(|\zeta|^2) \frac{d^2 \zeta}{\pi}, \quad (6)$$

where $d^2 z$ and $d^2 \zeta$ are the respective Lebesgue measures on the complex planes. The positive weight function $0 \leq t \mapsto \varpi_q(t)$ solves the following Stieltjes moment problem:

$$\int_0^\infty t^n \varpi_q(t) dt = x_n! = n! q^{\frac{n(n+1)}{2}}, \quad q \geq 1, \quad (7)$$

where $x_n \stackrel{\text{def}}{=} n! q^{\frac{n(n+1)}{2}}$ and we have adopted the generalized factorial notation as $x_n! \stackrel{\text{def}}{=} x_n x_{n-1} \cdots x_1, x_0! = 1$.

In the Hilbert space

$$L^2(\mathbb{C}^2, \mu(d\mathbf{x})) = L^2\left(\mathbb{C}, e^{-|z|^2} \frac{d^2 z}{\pi}\right) \otimes L^2\left(\mathbb{C}, \varpi_q(|\zeta|^2) \frac{d^2 \zeta}{\pi}\right)$$

we select the orthonormal set of functions

$$\Phi_{m,n}(\mathbf{x}) \stackrel{\text{def}}{=} \frac{\bar{z}^m}{\sqrt{m!}} \frac{\bar{\zeta}^n}{\sqrt{x_n!}}, \quad (8)$$

which we put in one-to-one correspondence with the elements $|m, n\rangle, m, n \in \mathbb{N}$, of an orthonormal basis of a separable Hilbert space \mathcal{H} . The states (8) obey a finite sum property for any $\mathbf{x} \in \mathbb{C}^2$:

$$\sum_{m,n \in \mathbb{N}} |\Phi_{m,n}(\mathbf{x})|^2 = e^{|z|^2} \mathcal{E}_q(|\zeta|^2) < \infty, \quad (9)$$

where $\mathcal{E}_q(t)$ is the generalized “exponential” built from the sequence x_n :

$$\mathcal{E}_q(t) = \sum_{n=0}^\infty \frac{t^n}{x_n!} = \sum_{n=0}^\infty q^{-\frac{n(n+1)}{2}} \frac{t^n}{n!}. \quad (10)$$

It is clear that, due to the condition $q \geq 1$, the convergence radius of this power series is infinite. On the other hand it is zero if $q < 1$. In the sequel we use the notation

$$\mathcal{N}(\mathbf{x}) \stackrel{\text{def}}{=} e^{|z|^2} \mathcal{E}_q(|\zeta|^2) .$$

We now introduce the CS corresponding to the above choice of orthonormal set. They are elements of \mathcal{H} defined by:

$$\begin{aligned} |z, \zeta\rangle \equiv |\mathbf{x}\rangle &\stackrel{\text{def}}{=} \frac{1}{\sqrt{\mathcal{N}(\mathbf{x})}} \sum_{m,n} \overline{\Phi_{m,n}(\mathbf{x})} |m, n\rangle \\ &= \frac{e^{-\frac{|z|^2}{2}}}{\sqrt{\mathcal{E}_q(|\zeta|^2)}} \sum_{m,n} \frac{z^m}{\sqrt{m!}} \frac{\zeta^n}{\sqrt{n!}} |m, n\rangle . \end{aligned} \quad (11)$$

By construction, these states are labeled by points of \mathbb{C}^2 and have three important properties [14]:

1. Continuity

The map $\mathbb{C}^2 \ni \mathbf{x} = (z, \zeta) \mapsto |z, \zeta\rangle \in \mathcal{H}$ is continuous.

2. Normalization

$$\langle z, \zeta | z, \zeta \rangle = 1 . \quad (12)$$

3. Resolution of the unity

The states form a continuous overcomplete set in \mathcal{H} which resolves the identity:

$$\int_{\mathbb{C}^2} \mu(d\mathbf{x}) \mathcal{N}(\mathbf{x}) |\mathbf{x}\rangle \langle \mathbf{x}| = \mathbb{I}_{\mathcal{H}} . \quad (13)$$

This resolution of the unity allows one to proceed to what we call CS quantization or Berezin-Klauder-Toeplitz quantization. It consists in mapping functions $f(\mathbf{x})$ (or even distributions) on \mathbb{C}^2 to operators A_f in \mathcal{H} given as continued superpositions of CS projectors weighted by f :

$$A_f \stackrel{\text{def}}{=} \int_{\mathbb{C}^2} \mu(d\mathbf{x}) f(\mathbf{x}) \mathcal{N}(\mathbf{x}) |\mathbf{x}\rangle \langle \mathbf{x}| . \quad (14)$$

A reasonable requirement on f or on considered distributions is that the mean value or *lower symbol* of A_f in CS, $\langle \mathbf{x} | A_f | \mathbf{x} \rangle$, should be smooth functions on \mathbb{C}^2 .

In the specific above formulated problem of the particle in a magnetic field (2), the value of q is fixed to $q = e^2 \approx 7.39$. The derivation [11] is based on the algebraic construction of CS for the motion of a particle on a circle given in [12]. Here we take the freedom to consider any value of q in the range $q \in [1, \infty)$.

Our aim is to find the weight function $\varpi_q(t)$ solving the Stieltjes moment problem (7) for any $q > 1$ since for $q = 1$ the solution is well known, $\varpi_1(t) = e^{-t}$, which, for a particle in a magnetic field, corresponds to the standard CS

of Malkin and Man'ko [13]. For that purpose, we will use a Mellin inverse transformation. But before, it is necessary to recall the conditions of existence of a Mellin inverse transform in the present situation.

Theorem 1 (see [15]) *Suppose that a function $\Phi(z)$ of the complex variable z , regular in the strip $S = \{z = \sigma + i\tau : a < \sigma < b\}$, is such that $\Phi(z) \rightarrow 0$ as $|\tau| \rightarrow \infty$ uniformly in the strip $a + \eta \leq \sigma \leq b - \eta$ for any arbitrarily small $\eta > 0$. If*

$$\int_{-\infty}^{+\infty} |\Phi(\sigma + i\tau)| d\tau < \infty \quad (15)$$

for each $\sigma \in (a, b)$ and if a function $\phi(x)$ is defined by

$$\phi(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-z} \Phi(z) dz \quad (16)$$

for $x > 0$ and a fixed $c \in (a, b)$, then we have the existence of $\Phi(z)$ as the Mellin transform of $\phi(x)$, namely

$$\Phi(z) = \int_0^{+\infty} \phi(x) x^{z-1} dx. \quad (17)$$

In the case under consideration, we have, with $z = \sigma + i\tau$,

$$\Phi(z) = \Gamma(z) q^{\frac{z(z-1)}{2}} = \Gamma(\sigma + i\tau) q^{\frac{\sigma(\sigma-1)}{2}} q^{i\tau \frac{2\sigma-1}{2}} q^{-\frac{\tau^2}{2}}.$$

From the boundedness property of the Gamma function,

$$|\Gamma(\sigma + i\tau)| = \left| \int_0^{+\infty} t^{\sigma-1} t^{i\tau} e^{-t} dt \right| \leq \int_0^{+\infty} t^{\sigma-1} e^{-t} dt = \Gamma(\sigma) < \infty \quad \forall \sigma > 0,$$

we infer the boundedness property for Φ :

$$|\Phi(z)| \leq \Gamma(\sigma) q^{\frac{\sigma(\sigma-1)}{2}} q^{-\frac{\tau^2}{2}}.$$

Hence, we can assert that

$$|\Phi(z)| \xrightarrow{\tau \rightarrow \infty} 0 \text{ for all } \sigma \in (a, b) \text{ with } a \geq 0.$$

Moreover, the condition

$$\int_{-\infty}^{+\infty} |\Phi(\sigma + i\tau)| d\tau \leq \Gamma(\sigma) q^{\frac{\sigma(\sigma-1)}{2}} \int_{-\infty}^{+\infty} q^{-\frac{\tau^2}{2}} d\tau = \sqrt{\frac{2\pi}{\ln q}} < \infty,$$

is clearly fulfilled for all $q > 1$. Let us now attempt to determine the function $\phi(x)$ formally defined for $x > 0$ by

$$\phi(x) = \frac{1}{2\pi} \int_{c-i\infty}^{c+i\infty} x^{-c-i\tau} \Gamma(c + i\tau) q^{\frac{c(c-1)}{2}} q^{i\tau \frac{2c-1}{2}} q^{-\frac{\tau^2}{2}} d\tau.$$

Choosing $c = 1$ for simplicity purpose, and writing $q = e^\lambda$, $\lambda > 0$, we obtain the integral

$$\phi(x) = \frac{1}{2\pi x} \int_{-\infty}^{+\infty} \left(\frac{x}{\sqrt{q}} \right)^{-i\tau} \Gamma(1+i\tau) e^{-\lambda \frac{x^2}{2}} d\tau.$$

It is legitimate to use here the integral representation of the Gamma function, $\Gamma(1+i\tau) = \int_0^{+\infty} t^{i\tau} e^{-t} dt$ and to invert the order of integration from the Fubini theorem. Hence,

$$\phi(x) = \frac{1}{2\pi x} \int_0^{+\infty} dt e^{-t} \int_{-\infty}^{+\infty} e^{-iy\tau} e^{-\lambda \frac{x^2}{2}} d\tau,$$

where we have introduced the variable $y \in \mathbb{R}$ such that $x = t\sqrt{q}e^y$. Performing the integral on τ which is the Fourier transform of a Gaussian, we obtain:

$$\phi(x) = \frac{1}{\sqrt{2\pi\lambda}x} \int_0^{+\infty} dt e^{-t} e^{-\frac{y^2}{2\lambda}} = \frac{1}{\sqrt{2\pi q \ln q}} \int_0^{+\infty} du e^{-\frac{x}{\sqrt{q}}u} e^{-\frac{(\ln u)^2}{2 \ln q}},$$

where we have introduced the new integration variable $u = \frac{\sqrt{q}}{x} t$.

We have eventually arrived at the following positive answer to our moment problem.

Proposition 2 *The solution of the moment problem*

$$\int_0^\infty t^n \varpi_q(t) dt = n! q^{\frac{n(n+1)}{2}}, \quad q \geq 1,$$

is given under the form of the following Laplace transform:

$$\varpi_q(t) = \frac{1}{\sqrt{2\pi q \ln q}} \int_0^{+\infty} du e^{-\frac{t}{\sqrt{q}}u} e^{-\frac{(\ln u)^2}{2 \ln q}} = \frac{1}{\sqrt{2\pi q \ln q}} \mathcal{L} \left[e^{-\frac{(\ln u)^2}{2 \ln q}} \right] \left(\frac{t}{\sqrt{q}} \right).$$

4 CS quantization of the complex plane

Since from the very beginning we deal with a tensor product of two Hilbert spaces and the resulting tensor products of standard Glauber-Sudarshan CS [16, 17] ($q = 1$) with non-standard CS ($q > 1$), let us examine the CS quantization of the complex plane produced by the latter, knowing that at the limit $q \rightarrow 1$ we get back the canonical quantization [18]. Let us start with the family of CS

$$\mathbb{C} \ni \zeta \mapsto |\zeta\rangle = \frac{1}{\sqrt{\mathcal{E}_q(|\zeta|^2)}} \sum_{n=0}^{+\infty} \frac{\zeta^n}{\sqrt{x_n!}} |n\rangle \in \mathcal{K}, \quad x_n = n q^n, \quad q \geq 1, \quad (18)$$

where $\{|n\rangle, n \in \mathbb{N}\}$ is an orthonormal basis of the separable Hilbert space \mathcal{K} , the latter being arbitrarily chosen. The CS quantization of the complex plane with these states is the map

$$f(\zeta, \bar{\zeta}) \mapsto \int_{\mathbb{C}} \frac{d^2\zeta}{\pi} \varpi_q(|\zeta|^2) f(\zeta, \bar{\zeta}) \mathcal{E}_q(|\zeta|^2) |\zeta\rangle \langle \zeta| \stackrel{\text{def}}{=} \hat{f}, \quad (19)$$

defining the operator \hat{f} in \mathcal{K} . Note that the complex plane is not equipped any more with its usual symplectic 2-form $\frac{i}{2\pi}d\zeta \wedge d\bar{\zeta}$, but instead is equipped with the weighted 2-form $\varpi(|\zeta|^2) \mathcal{E}_q(|\zeta|^2) \frac{i}{2\pi}d\zeta \wedge d\bar{\zeta}$.

For the simplest functions $f(\zeta, \bar{\zeta}) = \zeta$ and $f(\zeta, \bar{\zeta}) = \bar{\zeta}$ we obtain lowering and raising operators respectively,

$$\begin{aligned}\hat{\zeta} &= A, \quad A |n\rangle = \sqrt{x_n} |n-1\rangle, \quad A |0\rangle = 0, \quad \text{lowering operator}, \\ \hat{\bar{\zeta}} &= A^\dagger, \quad A^\dagger |n\rangle = \sqrt{x_{n+1}} |n+1\rangle, \quad \text{raising operator}.\end{aligned}\tag{20}$$

These operators obey the non-canonical commutation rule: $[A, A^\dagger] = x_{N+1} - x_N$, where x_N is defined by $x_N = A^\dagger A$ and is such that its spectrum is exactly $\{x_n, n \in \mathbb{N}\}$ with eigenvectors $x_N |n\rangle = x_n |n\rangle$. The linear span of the triple $\{A, A^\dagger, x_N\}$ is obviously not closed under commutation and the set of resulting commutators gives generically rise to an infinite Lie algebra. The fact that the complex plane has become non-commutative through this quantization is apparent from the quantization of the real and imaginary parts of $\zeta = (q + ip)/\sqrt{2}$:

$$\hat{q} = \frac{1}{\sqrt{2}} (A + A^\dagger), \quad \hat{p} = \frac{1}{\sqrt{2}i} (A - A^\dagger), \quad [\hat{q}, \hat{p}] = i(x_{N+1} - x_N). \tag{21}$$

Generally, given a function f on the complex plane, the resulting operator \hat{f} , if it exists, at least in a weak sense, acts on the Hilbert space \mathcal{K} with orthonormal basis $|n\rangle$: the integral

$$\langle \psi | \hat{f} | \psi \rangle = \int_{\mathbb{C}} \frac{d^2\zeta}{\pi} \varpi_q(|\zeta|^2) f(\zeta, \bar{\zeta}) \mathcal{E}_q(|\zeta|^2) |\langle \psi | \zeta \rangle|^2, \tag{22}$$

should be finite for any $\psi \in$ some dense subset in \mathcal{K} . One should notice that if ψ is normalized then (22) represents the mean value of the function f with respect to the ψ -dependent probability distribution $\zeta \mapsto |\langle \psi | \zeta \rangle|^2$ on the complex plane.

In order to be mostly rigorous on this important point, let us adopt the following acceptance criteria for a function to belong to the class of quantizable classical observables.

Definition 3 *A function $\mathbb{C} \ni \zeta \mapsto f(\zeta, \bar{\zeta}) \in \mathbb{C}$ is a CS quantizable classical observable along the map $f \mapsto \hat{f}$ defined by (19) if the map $\mathbb{C} \ni \zeta = \frac{1}{2}(q + ip) \equiv (q, p) \mapsto \langle \zeta | \hat{f} | \zeta \rangle$ is a smooth ($\sim \in C^\infty$) function with respect to the (q, p) coordinates of the complex plane.*

The function f is the *upper* [19] or *contravariant* [20] symbol of the operator \hat{f} , and $\langle \zeta | \hat{f} | \zeta \rangle$ is the *lower* [19] or *covariant* [20] symbol of the operator \hat{f} .

In [21] such a definition is extended to distributions with compact support on the plane.

Hence, localization properties in the complex plane from the point of view of the sequence $\{x_n\}_{n \in \mathbb{N}}$ should be examined from the shape (versus ζ) of the respective lower symbols

$$\check{q}(z) \stackrel{\text{def}}{=} \langle \zeta | \hat{q} | \zeta \rangle, \check{p}(\zeta) \stackrel{\text{def}}{=} \langle \zeta | \hat{p} | \zeta \rangle, \quad (23)$$

and the “noncommutative reading” of the complex plane should be encoded in the behavior of the lower symbol $\langle \zeta | [\hat{q}, \hat{p}] | \zeta \rangle$ of the commutator $[\hat{q}, \hat{p}]$. The study, within the above framework, of the product of dispersions

$$(\Delta_\zeta \hat{q}) (\Delta_\zeta \hat{p}) = \frac{1}{2} \langle \zeta | (x_{N+1} - x_N) | \zeta \rangle, \quad (24)$$

expressed in states $|\zeta\rangle$ should be thus relevant.

Let us now consider the CS quantized version of the classical harmonic oscillator Hamiltonian $H = \frac{1}{2} (p^2 + q^2) = |\zeta|^2$. We get the diagonal operator $\hat{H} = A A^\dagger$ with spectrum x_{N+1} . It is then natural to investigate the time evolution of the quantized version of the classical phase space point through its lower symbol:

$$\check{\zeta}(t) \stackrel{\text{def}}{=} \langle \zeta | e^{-i\hat{H}t} \hat{\zeta} e^{i\hat{H}t} | \zeta \rangle = \zeta \frac{1}{\mathcal{E}_q(|\zeta|^2)} \sum_{n=0}^{+\infty} \frac{|\zeta|^{2n}}{x_n!} e^{-i(x_{n+2} - x_{n+1})t}, \quad (25)$$

and to compare with the phase space circular classical trajectories, at different values of q . This study will be carried out in a forthcoming paper devoted to the noncommutative version of the model considered in this paper.

5 Final remarks

We have generalized the construction of coherent states for a particle in a magnetic field proposed in [11]. We have proved the resolution of the identity for the states by explicitly solving a specific version of the mathematical Stieltjes moment problem. So, using the CS, we are in position to fulfill the Berezin-Klauder-Toeplitz quantization.

As an application of this construction, in a future work, we will present the CS quantization of a charged particle in a uniform magnetic field in a non-commutative geometry. In addition we will investigate the dispersion relations and the time evolution of the CS quantized version of the system for different members of our family of CS, i.e., at different values of our parameter q .

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